3.7. Reference Level

By applying the buoyancy correction for the lower portion of the piston as a reference level change, eq (16) becomes

$$\Delta h = y_{fp} - \frac{V_{fp}}{A_e} \qquad (19)$$

where y_{fp}=the length of the piston below the cylinder,
V_{fp}=the volume of the piston below the cylinder.

4. Area

The effective area, A_e, of the piston, can be expressed by the relationship

$$A_e = A_0[1+a(t-t_s)](1+bp_p)[1+d(p_z-p_j)] \qquad (20)$$

where A_0=the effective area of the piston at temperature, t_s, and atmospheric pressure,
a=the fractional change in effective area per unit change in temperature,
t=the temperature of the piston and cylinder,
t_s=the reference temperature,
b=the fractional change in effective area per unit change in pressure,
d=the fractional change in effective area per unit change in jacket pressure of a controlled clearance piston gage,
p_z=the jacket pressure required to reduce the piston clearance to zero at pressure p_p,
p_j=the jacket pressure.

4.1. Temperature Coefficient of Area

The fractional change in effective area per unit change in temperature can be determined as follows:

$$a = \alpha_k + \alpha_c \qquad (21)$$

where α_k=the temperature coefficient of linear expansion of the piston,
α_c=the temperature coefficient of linear expansion of the cylinder.

The most convenient reference temperature, t_s, is the average temperature of the room in which the instrument is used. In many instances the difference $t-t_s$ may be insignificant.

The temperature, t, of the piston and cylinder is usually assumed to be the same as the temperature of the base of the instrument. The fact is, in practice, the piston and cylinder are usually at a temperature higher than that of the rest of the instrument, although, in a gas lubricated piston gage they may be lower. So many factors affect the temperature that the order of magnitude calculation of the temperature rise may be unreliable. Some of the more important factors are: speed of rotation, clearance, pressure, viscosity, Joule-Thompson coefficient, and thermal conductivity of the pressure fluid. One precaution that can be taken to keep the uncertainty of temperature, t, from being unnecessarily large, is to keep the speed of rotation no greater than is required to maintain hydrodynamic lubrication.

4.2. Effective Area at Atmospheric Pressure

A_0 is very nearly equal to the mean of the area of the piston and the area of the cylinder [3,4] at the reference temperature and can be calculated as follows:

$$A_0 = \frac{A_k + A_c}{2}[1+a(t_s-t_m)] \qquad (22)$$

where A_k=the area of the piston,
A_c=the area of the cylinder,
t_m=the temperature at which A_k and A_c are measured.

The value of $\frac{A_k+A_c}{2}$ may be determined from direct measurements of the diameters of the piston and cylinder, or by comparison with a piston gage of known area. In a controlled clearance piston gage, the jacket pressure, applied to the outside of the cylinder, is used to vary the diameter of the cylinder as desired and A_c is assumed to be equal to A_k.

4.3. Pressure Coefficient of Area

The fractional change in area with pressure is most readily obtained by comparing the instrument against a controlled clearance piston gage for which the pressure coefficient, b, can be computed, or by comparison with a piston gage of known characteristic. To the first approximation, for a controlled clearance piston gage,

$$b = \frac{3\mu - 1}{Y} \qquad (23)$$

where μ=Poisson's ratio for the piston,
Y=Young's modulus for the piston.

The product, bp_p, is small and therefore an approximate value for p_p is adequate. Deffet and Trappeniers [4], Dadson [6], Bridgman [7], and Ebert [8] give more detailed discussions of the elastic distortion of piston gages.

4.4. Change of Area With Jacket Pressure

The factor $[1+d(p_z-p_j)]$ is used only for controlled clearance piston gages. The fractional change of area with jacket pressure, d, is best obtained experimentally by varying the jacket pressure, p_j, and measuring the change in pressure, p_p. Since the change in pressure will be small, the measuring instrument must be sensitive.

The jacket pressure required to reduce the clearance between the piston and cylinder to zero, at a particular value of p_p, may be determined by two methods. One method is to observe the torque required to turn the piston as the jacket pressure is varied. An abrupt increase of torque is observed when the clearance is reduced to zero. The other method is to measure the fall rate of the piston at several jacket pressures. The cube root of the fall rate is then plotted against jacket pressure, and the curves are extrapolated to zero fall rate to get values of p_z.

5. Piston Gage Pressure

The complete equation for p_p can be obtained from combining eqs (2), (18), and (20);

$$p_p=\frac{\frac{M_m}{A_o}\left(1-\frac{\rho_a}{\rho_m}\right)kg_L+\frac{M_{fa}}{A_o}\left(1-\frac{\rho_a}{\rho_{fa}}\right)kg_L+\frac{\gamma C}{A_o}}{[1+a(t-t_s)](1+bp_p)[1+d(p_z-p_j)]} \tag{24}$$

where the reference level is determined as shown in eq (19).

Equation (24) would be rather formidable if it were necessary to solve it for each pressure measurement. Fortunately the terms can be grouped so that the amount of calculation can be reduced to practical proportions for some instruments, and some terms can be ignored if the accuracy requirements are low. It should be noted that eq (24) is not exact. Some second order terms have been dropped and the coefficients are constant only to a first approximation.

6. Conclusions

The accuracy of pressure measurements depends not only on the performance of the piston gage, but on the application of corrections derived from parameters of the system. These depend upon the construction of the instrument, composition of the pressure fluid, environment, pressure, and physical arrangement of the pressure system. The accuracy to which the values of these parameters are known usually establishes the overall accuracy of the measurements.

7. References

[1] The piston gage as a precise pressure measuring instrument, D. P. Johnson and D. H. Newhall, Trans. ASME **75**, 3, 301 (April 1953).

[2] Smithsonian Meteorological Tables, published by the Smithsonian Institution, Washington, D.C., Sixth Revised Edition (1951).

[3] A multiple manometer and piston gages for precision measurements, C. H. Meyers and R. S. Jessup, BS J. Research **6**, 1061 (June 1931).

[4] Les balance manometriques, L. Deffet and N. Trappeniers, Memorial de l'Artillerie Francaise, 4 fascicule (1954).

[5] Elastic distortion error in the dead weight piston gage, D. P. Johnson, J. L. Cross, J. D. Hill, and H. A. Bowman, Ind. Eng. Chem. **49**, 2046 (Dec. 1957).

[6] A new method for the absolute measurement of high pressure, R. S. Dadson, Nature **176**, 188 (July 30, 1955).

[7] The measurement of hydrostatic pressures up to 20,000 kilograms per square centimeter, P. W. Bridgman, Proc. Am. Acad. Arts and Sci. **47**, 321 (1912).

[8] Aufstellung einer Druckskale und deren experimentelle Erprobung bis 20,000 at., H. Ebert, Z. Angew. Phys. **7**, 331 (March 1949).

8. Appendix A. Computation of Pressure

8.1. Calculation of Correction Factors

To illustrate a method for simplifying the calculations, suppose that a particular piston gage is to be used in Room 131, MTL Bldg. at the National Bureau of Standards in Washington, D.C., to calibrate Bourdon gages. The local value of gravity g_L=980.10 gals, ρ_a will be assumed to have the value of the average density of air at this location, so ρ_a=0.00117 g/cm^3. The fluid being used in the instrument is aviation instrument oil for which the density, ρ_{fa}=0.862 g/cm^3 or 0.0321 lb/in.^{3} and the surface tension, γ=0.00017 lbf/in., at atmospheric pressure and 25 °C.

The piston gage is not a controlled clearance piston gage, therefore the factor $1+d(p_z-p_j)$ will be omitted. From direct measurements on the instrument, we find that C=1.964 in., y_{fa}=2.5 in., V_{fa}=1.525 in.^{3}, y_{fp}=1.625 in., and V_{fp}=0.2778 in.^{3}.

From a previous calibration against a controlled clearance piston gage, we have: A_0=0.13024 in.^{2} (at t_s=25 °C) and $b=1.48\times10^{-7}$ in.^{2}/in.^{2} psi. We also know that the piston is steel with the temperature coefficient, $\alpha_k=12\times10^{-6}$ in./in. deg C at 25 °C and the cylinder is brass with the temperature coefficient $\alpha_c=18.4\times10^{-6}$ in./in. degree C at 25 °C.