



FIG. 7. Stress rate—acceleration diagram of acceleration wave reflection from a stress-free boundary.

#### Collision of two acceleration waves

Let us consider the interaction of a right-running wave connecting the state  $(\dot{\sigma}_2, \ddot{x}_2)$  with the state  $(\dot{\sigma}_2, \ddot{x}_2)$  and a left-running wave connecting the state  $(\dot{\sigma}_2, \ddot{x}_2)$  with the state  $(\dot{\sigma}_3, \ddot{x}_3)$ . The characteristic lines for each of these waves are as shown in Fig. 6. State 2 must lie at the intersection of the left- and right-running wave characteristics, because this is the only point on both lines. This state satisfies (A1). After the wave interaction the waves adjacent to states 1 and 3 are propagating in the opposite direction to what they were previously, so the state 4 between them must lie on each of the dotted cross curves, as shown in the Fig. 6.

#### Collision of an acceleration wave with a contact surface

As with any acceleration wave, the incident wave can be represented by a straight line connecting states in the  $(\dot{\sigma}, \ddot{x})$  plane. The reflected wave lies on a line through  $(\dot{\sigma}, \ddot{x}_1)$  with slope  $\rho_{01}U_{N1}$  and the transmitted wave lies on a line through  $(\dot{\sigma}_0, \ddot{x}_0)$  with slope  $-\rho_{02}U_{N2}$ . Since both  $\dot{\sigma}$  and  $\ddot{x}$  are continuous across the contact surface, the state between the two waves corresponds to the point  $(\dot{\sigma}_2, \ddot{x}_2)$  where the transmitted- and reflected-wave curves intersect. This solution satisfies (A7) as, of course, it must.

#### Reflection of an acceleration wave from a free surface

As mentioned previously, the free-surface reflection problem is a special case of the contact surface inter-

action problem in which the second material is replaced by a void. In this case  $\dot{\sigma} = 0$  and  $\rho_{02}U_{N2} = 0$ , and we have the situation illustrated in Fig. 7. From the drawing we obtain the results (5.10) given previously.

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